



88067401

**MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1**

Thursday 2 November 2006 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- Answer all the questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.



Maximum marks will be given for correct answers. Where an answer is wrong, some marks may be given for correct method, provided this is shown by written working. Working may be continued below the box, if necessary. Solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer.

1. Let $x = 7.94$.

- (a) Calculate the value of $\frac{2x+1}{x^3}$.
- (b) (i) Give your answer correct to **three** decimal places.
(ii) Write your answer to (b)(i) as a percentage.
- (c) Give your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$.

Working:

Answers:

- (a) _____
- (b) (i) _____
(ii) _____
- (c) _____

2. Two logic propositions are given

p : *Dany goes to the cinema*

q : *Dany studies for the test.*

(a) Write in words the proposition

$$p \vee q.$$

(b) Given the statement s : “*If Dany goes to the cinema then Dany doesn’t study for the test*”.

(i) Write s in symbolic form.

(ii) Write in symbolic form the contrapositive of part (b)(i).

Working:

Answers:

(a)

(b) (i)

(ii)



3. Andrew invests 20 000 Swiss francs in a bank that offers a 2 % simple interest per year for 8 years.
- (a) Find the interest he has after these 8 years.

Philip invests 20 000 Swiss francs for 6 years in a bank at a nominal rate of 5 % interest **compounded quarterly**.

- (b) Find the **total amount** in Philip's account after these 6 years.

Working:

Answers:

(a) _____

(b) _____



4. Consider the geometric sequence $8, a, 2, \dots$ for which the common ratio is $\frac{1}{2}$.
- (a) Find the value of a .
 - (b) Find the value of the eighth term.
 - (c) Find the sum of the first twelve terms.

Working:

Answers:

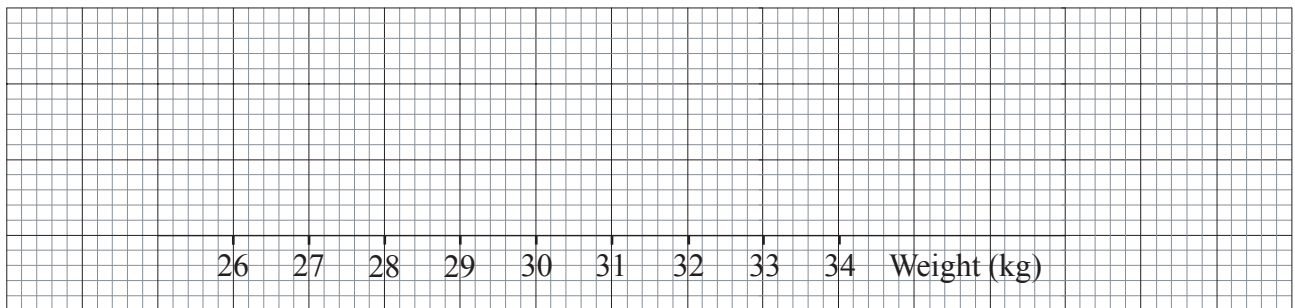
- (a) _____
- (b) _____
- (c) _____

5. The following stem and leaf diagram gives the weights in kg of 34 eight year-old children.

Stem	Leaf
26	1, 2
27	2, 4, 4
28	0, 1, 6, 6
29	2, 2, 4, 4, 5
30	0, 1, 2, <i>t</i> , 6, 8, 8, 9
31	3, 3, 5, 6, 6
32	1, 3, 5, 5, 8
33	0, 4

Key: 26|1 reads 26.1kg

- (a) The median weight is 30.3 kg. Find the value of *t*.
- (b) Write down the lower quartile weight.
- (c) The value of the upper quartile is 31.6 kg and there are no outliers. Draw a box and whisker plot of the data using the axis below.



Working:

Answers:

- (a) _____
- (b) _____
- (c) _____

6. (a) Expand the expression $2x(x^2 - 1)$.
- (b) Hence differentiate $f(x) = 2x(x^2 - 1)$ with respect to x .
- (c) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = -1$.
- (d) If the angle between the x -axis and the tangent in part (c) is θ , write down the value of $\tan \theta$.

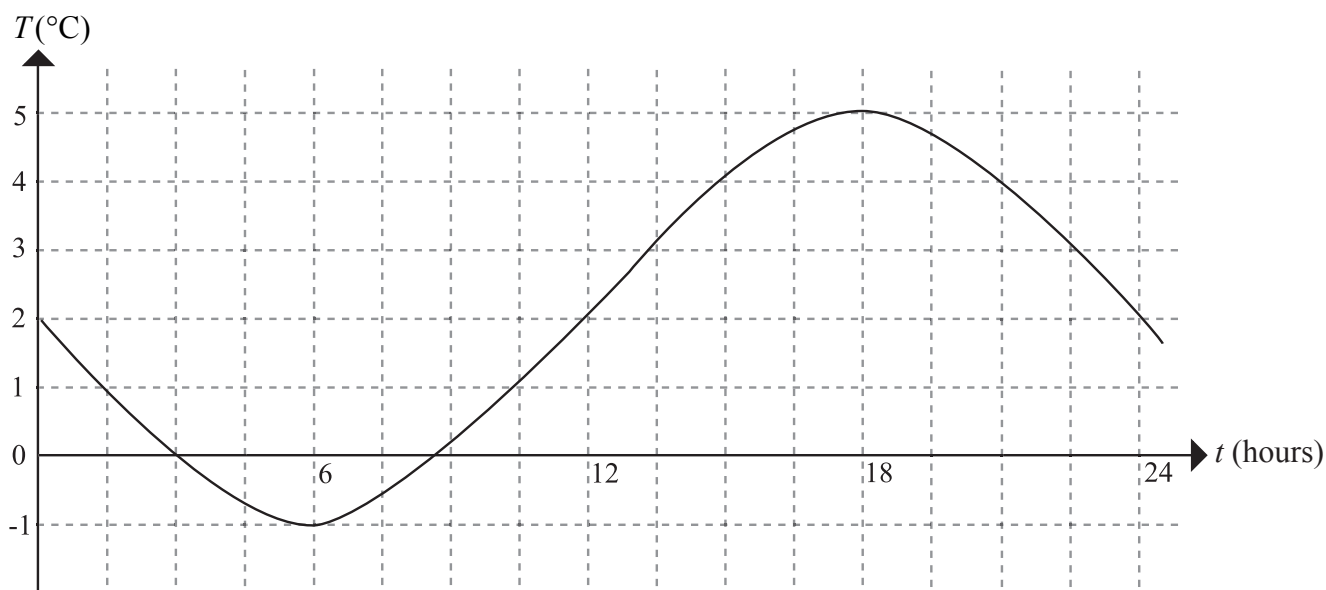
Working:

Answers:

- (a) _____
- (b) _____
- (c) _____
- (d) _____



7. The temperature ($^{\circ}\text{C}$) during a 24 hour period in a certain city can be modelled by the function $T(t) = -3 \sin(bt) + 2$, where b is a constant, t is the time in hours and bt is measured in degrees. The graph of this function is illustrated below.



- Determine how many times the temperature is exactly 0°C during this 24 hour period.
- Write down the time at which the temperature reaches its maximum value.
- Write down the interval of time in which the temperature changes from -1°C to 2°C .
- Calculate the value of b .

Working:

Answers:

- _____
- _____
- _____
- _____



8. The figure shows a triangular area in a park surrounded by the paths AB, BC and CA, where $AB = 400 \text{ m}$, $\hat{A}BC = 50^\circ$ and $\hat{B}CA = 30^\circ$.

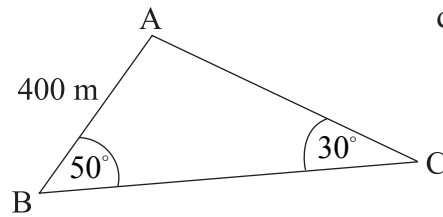


diagram not to scale

- (a) Find the length of AC using the above information.

Diana goes along these three paths in the park at an average speed of 1.8 m s^{-1} .

- (b) Given that $BC = 788 \text{ m}$, calculate how many minutes she takes to walk once around the park.

Working:

Answers:

- (a) _____
(b) _____

9. The graph of a quadratic function $f(x)$ intersects the horizontal axis at $(1, 0)$ and the equation of the axis of symmetry is $x = -1$.
- (a) Write down the x -coordinate of the other point where the graph of $y = f(x)$ intersects the horizontal axis.
- (b) $y = f(x)$ reaches its maximum value at $y = 5$.
- (i) Write down the value of $f(-1)$.
- (ii) Find the range of the function $y = f(x)$.

Working:

Answers:

- (a) _____
- (b) (i) _____
- (ii) _____

10. Oral tests are conducted by three examiners A, B and C separately. The results of the examination are classified as Credit, Pass or Fail. A χ^2 test is applied to the data collected in order to test whether or not the examiners differ in their standard of awards.

(a) State the null hypothesis, H_0 , for this data.

(b) Write down the number of degrees of freedom.

Of the 135 students who sit the exam, 30 get Credit and 45 are tested by examiner A.

(c) Calculate the expected number of students who get a Credit and are tested by examiner A.

Using a 5 % level of significance, the p -value is found to be 0.0327 correct to 3 s.f.

(d) State whether H_0 should be accepted. Justify your answer.

Working:

Answers:

(a)

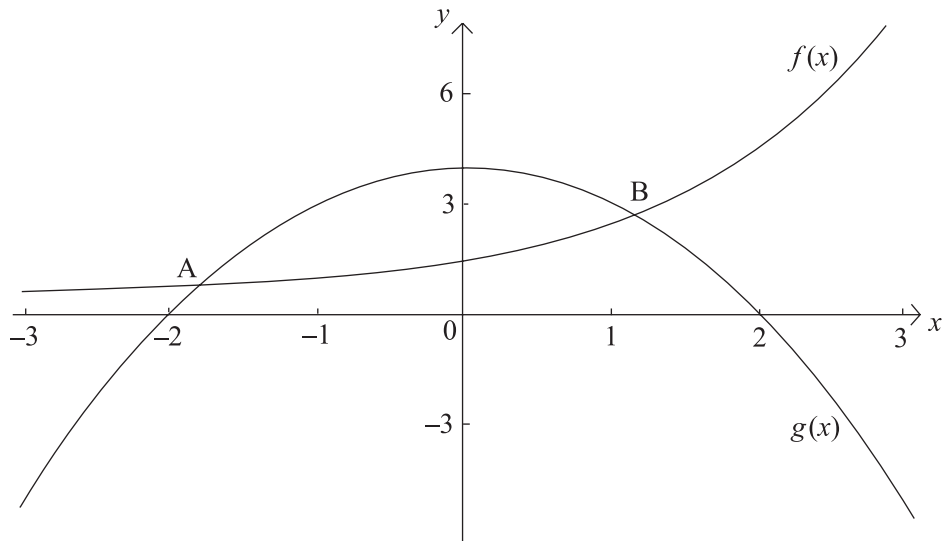
(b)

(c)

(d)



11. The figure below shows the graphs of the functions $f(x) = 2^x + 0.5$ and $g(x) = 4 - x^2$ for values of x between -3 and 3 .



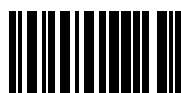
- (a) Write down the coordinates of the points A and B.
- (b) Write down the set of values of x for which $f(x) < g(x)$.

Working:

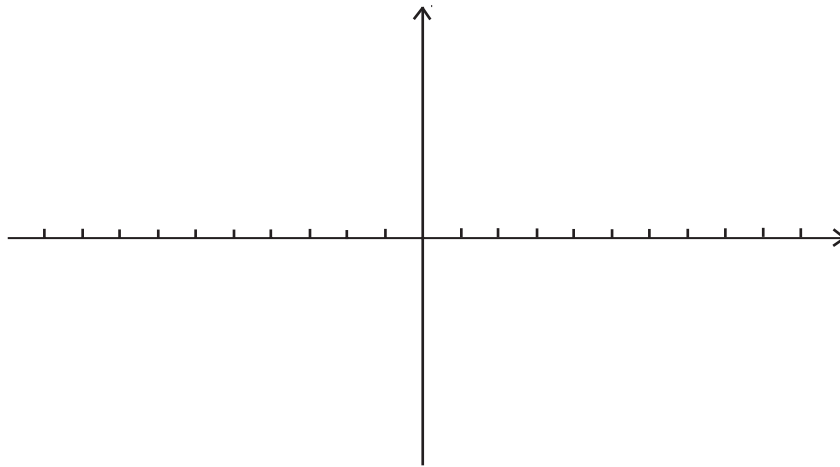
Answers:

(a)

(b)



12. (a) Sketch the graph of $y = 3 + \frac{3}{x-1}$ for $-10 \leq x \leq 10$.



- (b) Write down the equations of
- (i) the horizontal asymptote
 - (ii) the vertical asymptote.

Working:

Answers:

- (b) (i) _____
- (ii) _____



- 13.** Clara visits Britain from the United States and exchanges 1000 US dollars (USD) into pounds (GBP). The exchange rate is 1 USD = 0.543 GBP. The bank charges 2 % commission for each transaction.

(a) Calculate how many GBP she receives.

Next Clara wants to travel to France. She changes 150 GBP to euros (€) at a rate of 1 GBP = 1.35 €. The bank charges commission and then gives Clara 200 €.

(b) Find the amount of commission in GBP.

Working:

Answers:

(a)

(b)



14. The value of a car decreases each year. This value can be calculated using the function

$$v = 32\,000r^t, t \geq 0, 0 < r < 1,$$

where v is the value of the car in USD, t is the number of years after it was first bought and r is a constant.

- (a) (i) Write down the value of the car when it was first bought.
- (ii) One year later the value of the car was 27200 USD. Find the value of r .
- (b) Find how many years it will take for the value of the car to be less than 8000 USD.

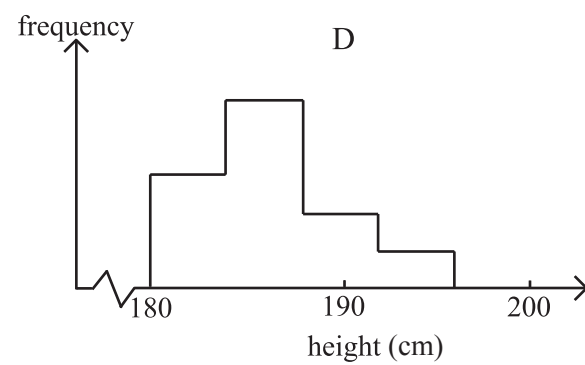
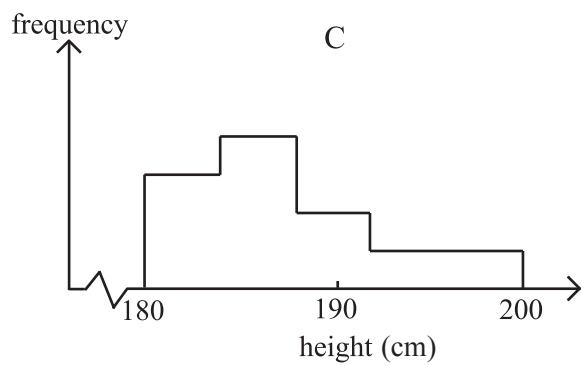
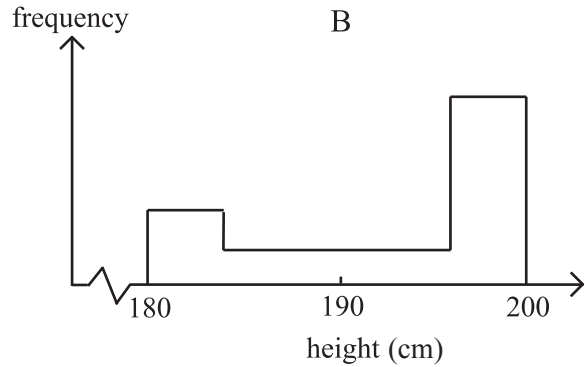
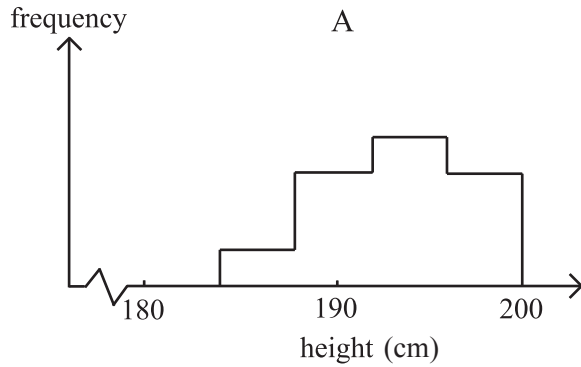
Working:

Answers:

- (a) (i) _____
- (ii) _____
- (b) _____



15. The heights in cm of the members of 4 volleyball teams A, B, C and D were taken and represented in the frequency histograms given below.



The mean \bar{x} and standard deviation σ of each team are shown in the following table.

	I	II	III	IV
\bar{x}	194	189	188	195
σ	6.50	4.91	3.60	3.74

(This question continues on the following page)



(Question 15 continued)

Match each pair of \bar{x} and σ (I, II, III, or IV) to the correct team (A, B, C or D).

\bar{x} and σ	Team
I	
II	
III	
IV	

Working: